EJECTION IN WIND TUNNELS.
THE THEORY EXPLAINING PRESSURE LOSSES IN SUBSONIC DIFFUSERS

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When testing an aircraft model in a wind tunnel, the drag of the model itself requires only a low proportion of the wind tunnel compressor power – say, the blockage ratio, which is a few per cent. An almost equal proportion is lost in the return duct. And the balance, i.e., 90% of the electric power consumed, is spent to recover the total pressure loss in wind tunnel diffusers. Therefore, it is crucially important to reveal mechanisms of the diffuser loss and predict it. Gas flow in a divergent diffuser is essentially multidimensional, viscous, often nonstationary and depends on flow nonuniformity at the diffuser entry station; see [1–9]. These difficulties did not allow researchers to identify major processes in a diffuser with an (almost) optimum divergence angle and develop a rather clear, one-dimensional loss theory which could describe flow with general initial nonuniformity. Crocco in [2] had cast doubt on even the possibility of creating any one-dimensional theory for analyzing diffusers; nonetheless, he appreciated one-dimensional approximations and had succeeded in this area.

Let us assume the diffuser internal flow to be composed of two areas: the core and the stagnant layer. The latter may be, firstly, the boundary layer over diffuser walls, and secondly, any stopped gas domains within the flow (such as a model wake in a wind tunnel diffuser). We can adopt the following assumptions:

– within each domain the gas parameter distribution is uniform over any cross section in the diffuser;
– static pressure in the stagnant layer is equal to static pressure in the core and increasing downstream;
– the gas flow rate in the stagnant domain is insignificant in comparison with the core gas flow rate;
– the mean speed in the stagnant layer is zero; and
– gas features no friction and heat exchange with diffuser walls.

The simplified physical model is depicted in Fig. 1; it is very similar to the flow pattern in a divergent mixing chamber of an ejector when the ejection coefficient is zero, so the present approach may be named "an ejection-based theory".

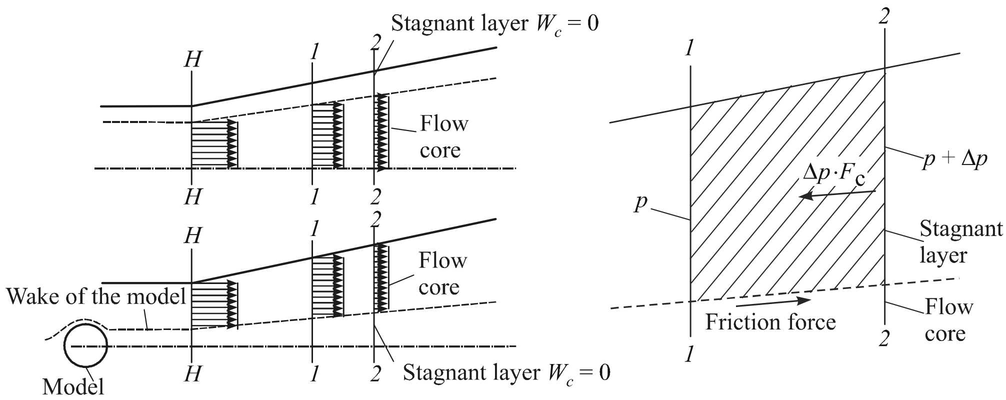

Figure 1.

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Report Documentation Page

Report Date	Report Type	Dates Covered (from... to)
23 Aug 2002	N/A	-

Title and Subtitle Ejection in Wind Tunnels, The Theory Explaining Pressure Losses in Subsonic Diffusers	**Contract Number**
	Grant Number
	Program Element Number
Author(s)	**Project Number**
	Task Number
	Work Unit Number
Performing Organization Name(s) and Address(es) Institute of Theoretical and Applied Mechanics Institutskaya 4/1 Novosibirsk 530090 Russia	**Performing Organization Report Number**
Sponsoring/Monitoring Agency Name(s) and Address(es) EOARD PSC 802 Box 14 FPO 09499-0014	**Sponsor/Monitor's Acronym(s)**
	Sponsor/Monitor's Report Number(s)

Distribution/Availability Statement
Approved for public release, distribution unlimited

Supplementary Notes
See also ADM001433, Conference held International Conference on Methods of Aerophysical Research (11th) Held in Novosibirsk, Russia on 1-7 Jul 2002

Abstract

Subject Terms

Report Classification unclassified	**Classification of this page** unclassified
Classification of Abstract unclassified	**Limitation of Abstract** UU

Number of Pages
5

For the gas portion between any two cross sections *1* and *2* we can write the mass equation, energy equation, and momentum variation equation. For the inverse problem of aerodynamics the layer thickness variation along the longitudinal axis can be written as

$$\frac{F_c}{F} = \overline{F}_c(\lambda) = H + \Delta H\left(1 - \frac{\lambda}{\lambda_{\mathrm{H}}}\right) \text{ or } \overline{F}_c(\lambda) = A - B\lambda$$

(where H is the relative thickness at the initial section, $Í+\Delta H$ is the thickness at infinity, and λ is the relative speed). With this in mind, the differential equation of motion becomes

$$\frac{dF_{\mathrm{core}}}{F_{\mathrm{core}}} = \frac{\lambda^2 - 1 - (A - B\lambda)\dfrac{2\kappa}{\kappa+1}\lambda^2}{\lambda\left(1 - \dfrac{\kappa-1}{\kappa+1}\lambda^2\right)} d\lambda$$

and may be integrated to produce

$$\ln F_{\mathrm{core}} + \ln C = \ln\left\{\frac{\left(1 - \dfrac{\kappa-1}{\kappa+1}\lambda^2\right)^{\frac{\kappa A}{\kappa-1}}}{\lambda\left(1 - \dfrac{\kappa-1}{\kappa+1}\lambda^2\right)^{\frac{1}{\kappa-1}}} \left[\frac{1 + \sqrt{\dfrac{\kappa-1}{\kappa+1}}\lambda}{1 - \sqrt{\dfrac{\kappa-1}{\kappa+1}}\lambda}\right]^{\frac{\kappa}{\kappa-1}\sqrt{\frac{\kappa+1}{\kappa-1}}\cdot B} \cdot e^{-\frac{2\kappa}{\kappa-1}B\lambda}\right\}.$$

Let us introduce a new gas dynamic function

$$\varphi(\lambda) = \left(\frac{1 + \sqrt{\dfrac{\kappa-1}{\kappa+1}}\lambda}{1 - \sqrt{\dfrac{\kappa-1}{\kappa+1}}\lambda}\right)^{\frac{\kappa}{\kappa-1}\sqrt{\frac{\kappa+1}{\kappa-1}}} \cdot e^{-\frac{2\kappa}{\kappa-1}\lambda}$$

and use the initial values to derive the integration constant and interrelate geometric and gas dynamic parameters of the diffuser as

$$\overline{F}_{\mathrm{core}} = \frac{F^{\mathrm{core}}}{F_{\mathrm{H}}^{\mathrm{core}}} = \frac{q(\lambda_H)}{q(\lambda)}\left[\frac{P(\lambda)}{P(\lambda_{\mathrm{H}})}\right]^A \cdot \left[\frac{\varphi(\lambda)}{\varphi(\lambda_{\mathrm{H}})}\right]^B = \frac{q(\lambda_{\mathrm{H}})}{q(\lambda)} \cdot \frac{1}{\nu}; \quad \overline{F} = \frac{F}{F_{\mathrm{H}}} = \frac{F^{\mathrm{core}}}{F_{\mathrm{H}}^{\mathrm{core}}} \cdot \frac{1 - (A - B\lambda_{\mathrm{H}})}{1 - (A - B\lambda)};$$

$$\nu = \frac{P_0}{P_{0\mathrm{H}}} = \left[\frac{P(\lambda_{\mathrm{H}})}{P(\lambda)}\right]^A \cdot \left[\frac{\varphi(\lambda_{\mathrm{H}})}{\varphi(\lambda)}\right]^B; \quad \overline{P} = \frac{P}{P_{\mathrm{H}}} = \nu\frac{P(\lambda)}{P(\lambda_{\mathrm{H}})}.$$

Here, the subscript "H" is to parameters at the initial section, ν is the diffuser total pressure recovery factor, F and F_{core} are cross-sectional areas of the diffuser and the flow core, and $q(\lambda)$ and $P(\lambda)$ are usual gas dynamics functions.

If the diffuser has no stagnation domains ($\grave{A}\equiv\hat{A}\equiv 0$), then the factor ν equals unity, and diffuser flow is isentropic. Figure 2 demonstrates dependence of the diffuser total pressure recovery factor on the undimensionalized cross section area $\overline{F}$ at Mach number M_{H}=0.8 and the maximum possible increment of the stagnant layer thickness, ΔH=0.3. It is seen that diffuser total pressure recovery factor gets lower if H and/or the diffuser extension ratio $\overline{F}$ is increased. Thus, the analysis shows that total pressure loss takes place in even a diffuser with no wall friction and no flow separation from walls. The loss occurs because the diffuser includes low-speed

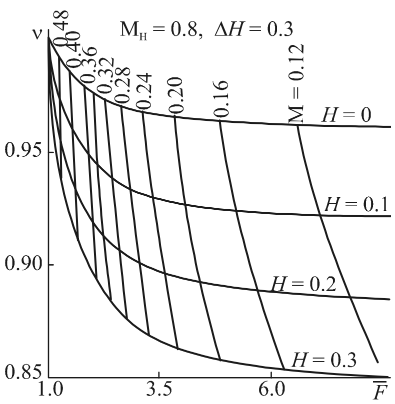

Figure 2

areas, and the loss immediately depends on sizes of the areas. Any undertakings diminishing the stagnant areas (for example, by boundary layer suction) always improve diffuser characteristics.

To understand the mechanisms decreasing total pressure in diffusers and increasing their entropy, we may again address the diffuser flow schematic in Fig. 1,*b*. The stagnant domain is subjected to the longitudinal gradient of static pressure and, thus, to the force that acts against the flow. For the domain to be in equilibrium (so that the gas is not flowing into the diffuser), the core should apply to the domain a force directed downstream. Such a force is provided by friction between the core and stagnant layer. The force generates the kinetic energy dissipation (at $\Delta H = 0$):

$$\overline{E}_{\partial} = \frac{E_{\text{diss}}}{E_{\text{kin}}} = \frac{\int_{F_H}^{F} F_c W dP}{m\dfrac{W_{\text{H}}^2}{2}} = H\left(1 - \lambda^2 / \lambda_{\text{H}}^2\right)$$

The relative dissipation is proportional to the stagnant layer thickness; and computation shows that the dissipation is almost equal to the relative energy of total pressure recovery, E_B in Fig. 3. Thus, all dissipation in the present theory is associated with friction between the core and stagnant layer; and there are no additional mechanisms that could increase flow entropy.

Most contributions to diffuser flow theories note that a considerable zone in a diffuser features an intense mixing of the flow by vortices which appear near walls. Also, many authors notice that the intense vortex formation is the primary factor in total pressure loss within subsonic diffusers.

As shown in Fig. 1,*b*, the static pressure differential applied to the elementary volume of the stagnant layer is equilibrated by the friction force which is at a distance of the stagnant layer half-thickness, thus forming a pair whose moment rotates the elementary volume around an axis normal to the drawing. This moment is the mechanism that turbulizes diffuser flow and causes the dissipation above.

Let us consider the simplest situation – a flat diffuser with a constant relative thickness of the stagnant layer. Determine the total turbulizing moment applied to the stagnant layer. The elementary volume (in Fig. 1,*b*) is subjected to the moment

$$dM = dP \cdot F_c \frac{1}{2} h_c = \frac{1}{2} \overline{F} H F_{\text{H}} h_{\text{H}} H \overline{F} dP = \frac{1}{2} F_{\text{H}} h_{\text{H}} P_{\text{H}} H^2 \overline{F}^2 d\overline{P}$$

where F_{H} and h_{H} are the diffuser section area and height at the initial station, $H = \dfrac{F_c}{F}$ is the relative area of the stagnant layer. The total turbulizing moment is

$$M_{\text{T}} = \int_{F_H}^{F} \frac{1}{2} F_{\text{H}} h_{\text{H}} P_{\text{H}} H^2 \overline{F}^2 d\overline{P} = \frac{1}{2} F_{\text{H}} h_{\text{H}} P_{\text{H}} H^2 \int_{F_H}^{F} \overline{F}^2 d\overline{P} .$$

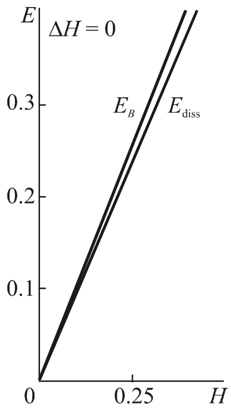

Figure 3

By introducing $M_T^0 = F_H P_H \frac{1}{2} h_H$, we write the relative turbulizing moment as

$$\overline{M}_T = \frac{M_T}{M_T^0} = H^2 \int\limits_{F_H}^{F} \overline{F}^2 d\overline{P} = \frac{2\kappa}{\kappa+1} H^2 (1-H) \frac{[q(\lambda_H)]^2}{[P(\lambda_H)]^{1+H}} \int\limits_{\lambda}^{\lambda_H} \frac{d\lambda}{\lambda\left(1-\frac{\kappa-1}{\kappa+1}\lambda^2\right)^{\frac{1-\kappa H}{\kappa-1}}}.$$

Of interest is a particular situation with H=1/7 (that is, where the stagnant layer occupies one-seventh of the diffuser cross section), which produces the denominator exponent of 2 and makes it possible to evaluate the integral analytically:

$$\overline{M}_T = \frac{\kappa}{\kappa+1} \cdot \frac{1}{7^2}\left(1-\frac{1}{7}\right) \frac{[q(\lambda_H)]^2}{[P(\lambda_H)]^{1+\frac{1}{7}}} [\psi(\lambda_H) - \psi(\lambda)],$$

where

$$\psi(\lambda) = \frac{1}{2\left(1-\frac{\kappa-1}{\kappa+1}\lambda^2\right)} + \frac{1}{2}\ln\left|\frac{\lambda^2}{1-\frac{\kappa-1}{\kappa+1}\lambda^2}\right|.$$

Line *1* in Fig. 4 represents the turbulizing moment dependence on the diffuser output speed at Mach number M_H=1; and line *2*, at a low value of M_H. The greater the diffuser expansion ratio, the greater the total turbulizing moment; so vortex formation is intense, and the flow gets entirely turbulized before reaching the diffuser output station.

It is interesting to consider application of the ejection-based theory to the case of incompressible fluid. Assume again that the relative thickness of the stagnant layer increases linearly, $\overline{F}_c = \frac{F_c}{F} = H + \Delta H\left(1-\frac{W}{W_H}\right)$. Then the pressure loss factor (or resistance coefficient) is $\zeta = H\left(1-\frac{W^2}{W_H^2}\right) + \Delta H\left(\frac{1}{3} - \frac{W^2}{W_H^2} + \frac{2}{3}\frac{W^3}{W_H^3}\right)$. It may be written very simply in terms of stagnant layer parameters and the input-to-output speed ratio. In the case of full stop (W=0) the latter equation becomes $\zeta = H + \frac{1}{3}\Delta H$; and if, in addition, the stagnant layer thickness is constant along the diffuser ($\Delta H = 0$), then the pressure loss factor equals the relative thickness of the stagnant layer: $\zeta = H = \frac{F_c}{F}$. This result is the foundation for understanding the performance features, computing the drag, and designing a diffuser with a nonuniform entry speed distribution (which is the case downstream of a wind tunnel test section).

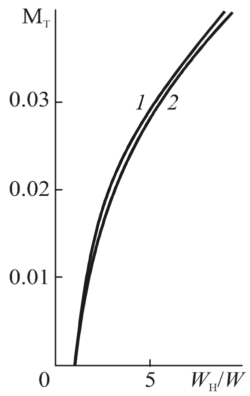

Figure 4

Looking at Fig. 5, it is clear that the theoretical total pressure recovery factor ν compares well with experimental values. For instance, data from [6] (by Bedrzhitsky) obtained in an installation with no boundary layer at the diffuser entry are close to the theory at $H = 0$. Experimental results by Lyzhin and Pasova in [9] were obtained in a

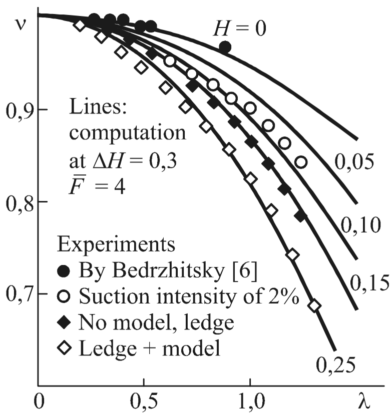

Figure 5

wind tunnel with no model in the test section upstream of the diffuser; these are close to the theoretical line at $H = 0.1$. When the test section is operated with free circulation and an exit ledge, the diffuser loss factor gets increased and corresponds to calculation at $H = 0.15$. When the test section includes a large model of an airplane (whose wing area is 20% of the test section area) at 20-degree incidence, the loss factor additionally increases and coincides with the theory at $H = 0.25$. Indeed, the model in the separated flow feature a wake whose cross-sectional area is about 0.1 of the test section area, and the wake is added to the boundary layer and the ledge wake.

Thus, the wind tunnel diffuser loss factor may at $M = 0$ be described by the equation $\zeta = H\left(1 - \frac{W^2}{W_H^2}\right) + 0{,}3\left(\frac{1}{3} - \frac{W^2}{W_H^2} + \frac{2}{3}\frac{W^3}{W_H^3}\right)$ – or $\zeta = 0{,}1+H$ in the case of a long diffuser (W=0). Transonic flows can be evaluated by using $\nu = \frac{P_0}{P_{0H}} = \left[\frac{P(\lambda_H)}{P(\lambda)}\right]^A \cdot \left[\frac{\varphi(\lambda_H)}{\varphi(\lambda)}\right]^B$, where A=0.3+H, B=0.3/λ, and H is the ratio of the cross-sectional area of the stagnant layer to that of the diffuser entry.

The major cause of pressure loss in both the ejector and the diffuser performance theory is collision of jets. The very idea to describe the diffuser loss in terms of collision energy coefficients was proposed and actively developed by Abramovich and Idel'chik in [1, 8]. The latter may be regarded as predecessors of the present theory, although diffuser loss could be compared with pressure loss in a flow with a ledge in only the case of very large expansion angles of diffusers. In usual diffusers of wind tunnels (such as those considered in [10]) the main stream collides with its own stopped (boundary) layer which cannot overcome the significant positive pressure gradient. And it is of no importance whether the stream thereafter continues moving at a low speed in the same direction (as in a pre-separation diffuser) or slowly moves inversely (as in the case of micro-separation) – the work of friction and the work of pressure loss for producing the gradient are almost identical.

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